More Compact Ferrite Circulator Junctions with Predicted Performance

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Abstract—**The analysis of two novel types of 3-port junction circulator is discussed in detail, and, neglecting losses, graphs of the predicted performance of specific theoretical designs are shown. The first type of circulator is based on a ferrite ring, and it is shown that by appropriately selecting the ratio of the inner/outer radius of the ring the need for transformers can be eliminated. An 11 GHz design with 1 GHz bandwidth is presented. The second type is based on a three-port ferrite disk having two ports positioned 60° away from a middle port. This provides a more compact layout and a design for 94 GHz is presented.**

Introduction

Symmetrical three-port microstrip and stripline circulators based on ferrite disks have been extensively treated in the literature [1]–[7]. In 1974 Wu and Rosenbaum [1] introduced a method for the design of these circulators which offer wide bandwidths. Characteristics of ring circulators have been investigated before [8]–[14]. Circulation conditions of symmetrical three-port ring circulators have been derived by Davis and Dimitriyev [10] but the performance of such circulators has not been discussed. These circulators have a smaller bandwidth than the disk type but a larger strip impedance is needed to couple to the ring. This may obviate the need for transformers. Asymmetrical W disk circulators in which ports 1 and 3 are 60° away from port 2 may offer convenient layout, but at the expense of deteriorating the bandwidth. The circulation conditions are presented here for the first time, and a design for 94 GHz and the predicted performance are discussed. In the following analysis, all losses are neglected [18].

Ring Circulators

The central conductor configuration of the three-port stripline or microstrip ring circulator is shown in Fig. 1. Symmetrical junctions to be treated in this paper could be obtained by setting $\phi_1 = 0$, $\phi_2 = 2\pi/3$, $\phi_3 = 4\pi/3$, $\psi_1 = \psi_2 = \psi_3 = \psi$, or $W_i = W$ where

$$W_i = 2R \sin(\psi_i) \qquad i = 1, 2, 3. \tag{1}$$

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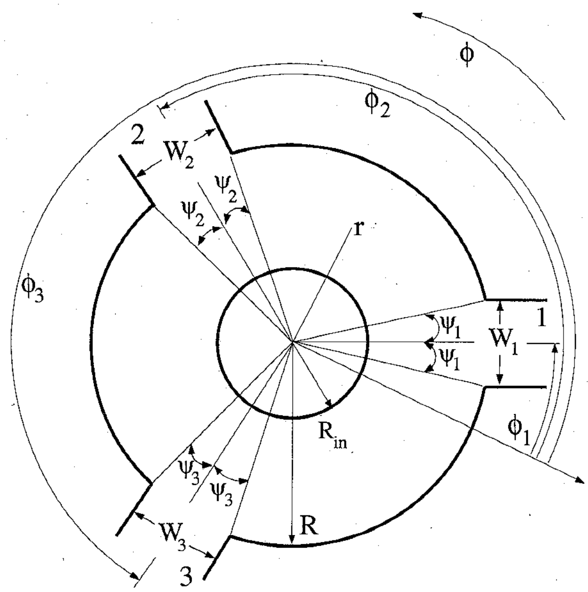

Fig. 1. Coordinate system for a ferrite ring Y circulator.

The magnetized ferrite ring(s) filling the space between the central ring conductor and the ground plate(s) have an outer radius R, an inner radius R_{in}, a relative dielectric permittivity ϵ_f, and an effective relative permeability μ_{eff}. Davis and Dimitriyev [10], [11] assumed that the inner curved circumference of the ring exhibits a magnetic wall as well as the outer one. For perfect circulation between two ports to occur the remaining port must be completely isolated, therefore transmission from port 1 to port 2 is achieved by setting the reflection coefficient at port 1 to zero or the input impedance at port 1 to unity. Since Z_{in_1} is a complex quantity two circulation conditions therefore arise as follows:

$$\text{Imaginary}(Z_{in_1}) = 0 \tag{2}$$

referred to as the first circulation condition, and

$$\text{Real}(Z_{in_1}) = 1 \tag{3}$$

referred to as the second circulation condition, where [10]

$$Z_{in_1} = Z_{11} - \frac{Z_{12}Z_{31}}{Z_{32}} \tag{4}$$

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The impedance matrix elements for the Y ring circulator have been derived [10] and are given by

$$Z_{li} = \frac{j\eta\sqrt{\psi_l\psi_i}}{\pi}\sum_{n=-\infty}^{\infty}\left\{\frac{\sin(n\psi_l)\sin(n\psi_i)}{n^2\psi_l\psi_i\Delta}[J_n(x_2)B_n(x_1) - Y_n(x_2)A_n(x_1)]\right\}\cdot e^{jn(\phi_i-\phi_l)} \quad \text{for } \mu_{\text{eff}} > 0 \tag{5}$$

and

$$Z_{li} = -\frac{j\eta\sqrt{\psi_l\psi_i}}{\pi}\sum_{n=-\infty}^{\infty}\left\{\frac{\sin(n\psi_l)\sin(n\psi_i)}{n^2\psi_l\psi_i\Delta_1}[I_n(x_2)P_n(x_1) - K_n(x_2)L_n(x_1)]\right\}\cdot e^{jn(\phi_i-\phi_l)} \quad \text{for } \mu_{\text{eff}} < 0 \tag{6}$$

and those for the disk (s = 0) by

$$Z_{li} = \frac{j\eta\sqrt{\psi_i\psi_l}}{\pi}\sum_{n=-\infty}^{\infty}\left\{\frac{\sin(n\psi_l)\sin(n\psi_i)}{n^2\psi_l\psi_i A_n(x_2)}\cdot J_n(x_2)\right\} e^{jn(\phi_i-\phi_e)} \quad \text{for } \mu_{\text{eff}} > 0 \tag{7}$$

and

$$Z_{li} = -\frac{j\eta\sqrt{\psi_i\psi_l}}{\pi}\sum_{n=-\infty}^{\infty}\left\{\frac{\sin(n\psi_l)\sin(n\psi_i)}{n^2\psi_l\psi_i L_n(x_2)}\cdot I_n(x_2)\right\} e^{jn(\phi_i-\phi_l)} \quad \text{for } \mu_{\text{eff}} < 0 \tag{8}$$

where

$$\mu_{\text{eff}} = \mu\left(1 - \left(\frac{\kappa}{\mu}\right)^2\right) \tag{9}$$

$$k_{\text{eff}} = \frac{2\pi f\sqrt{\mu_{\text{eff}}\epsilon_f}}{c}, \quad \eta = \frac{Z_{\text{eff}}}{Z_d} \tag{10}$$

$$x_2 = k_{\text{eff}}R \tag{11}$$

$$x_1 = x_2 s \tag{12}$$

$$s = \frac{R_{in}}{R} \tag{13}$$

$$A_n(x) = J_n'(x) + \frac{n\kappa}{\mu x}J_n(x) \tag{14}$$

$$B_n(x) = Y_n'(x) + \frac{n\kappa}{\mu x}Y_n(x) \tag{15}$$

$$\Delta = A_n(x_2)B_n(x_1) - A_n(x_1)B_n(x_2) \tag{16}$$

$$L_n(x) = I_n'(x) + \frac{n\kappa}{\mu x}I_n(x); \tag{17}$$

$$P_n(x) = K_n'(x) + \frac{n\kappa}{\mu x}K_n(x); \tag{18}$$

$$\Delta_1 = L_n(x_2)P_n(x_1) - L_n(x_1)P_n(x_2). \tag{19}$$

and κ and μ are the elements of the permeability tensor, $J_n(x)$ and $Y_n(x)$ are Bessel functions of the first and second kind respectively, $I_n(x)$ and $K_n(x)$ are Modified Bessel functions of the first and second kind respectively and the prime sign $'$ denotes the first derivative with respect to x.

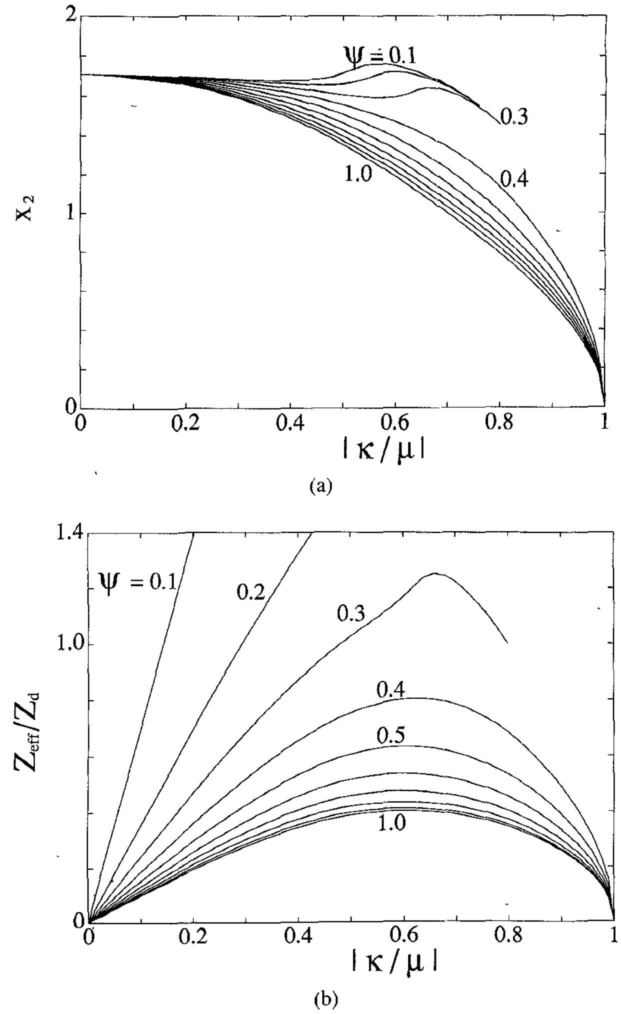


Fig 2. Circulation conditions of the Y ring with $s = 0.2$. (a) First condition. (b) Second condition.

The relative permeability tensor elements of the ring with an internal magnetic field well below that required for resonance at the design frequency of operation ($f_0 << f$) are

$$\kappa = -\frac{f_m}{f} \tag{20}$$

$$\mu = 1 \tag{21}$$

where

$$f_m = 2.8(4\pi M_s) \tag{22}$$

$$f_0 = 2.8H_o \tag{23}$$

$4\pi M_s$ is the magnetization saturation of the ferrite expressed in Gauss, and H_o is the internal magnetic field expressed in Oersted and assumed to be zero in this paper.

The circulation conditions for the main mode of circulation, (called Mode 1) for several values of s have been obtained where only terms up to fifth order have been retained as suggested by Davis & Dimitriyev [10], and only those for $s = 0.2$, 0.36 and 0.5, are shown in Figs. 2, 3,

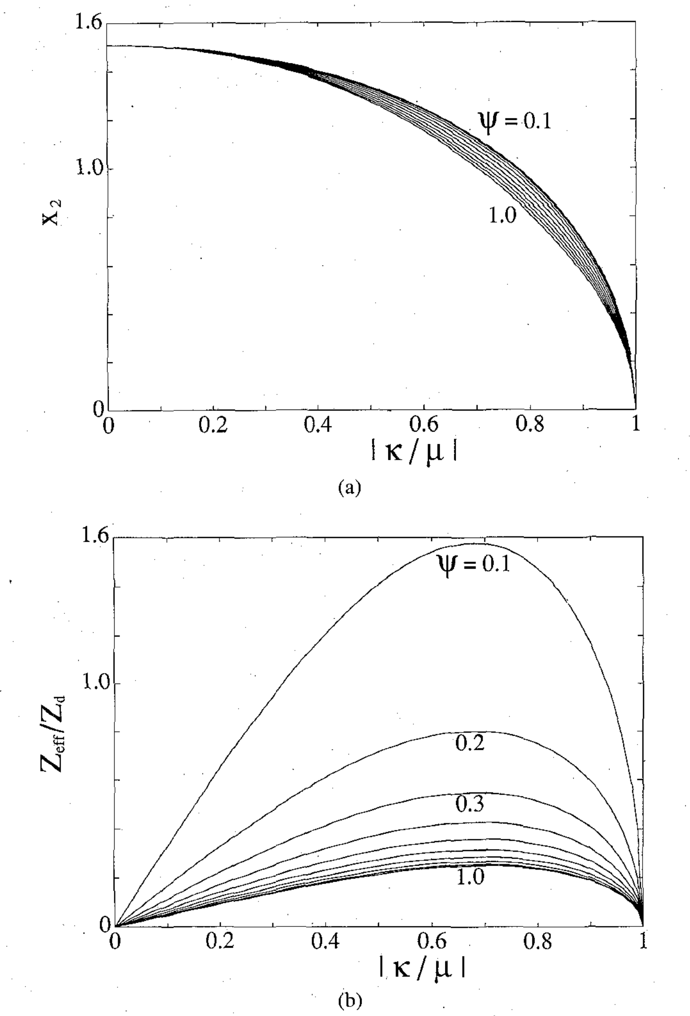


Fig. 3. Circulation conditions of the Y ring with $s = 0.36$. (a) First condition. (b) Second condition.

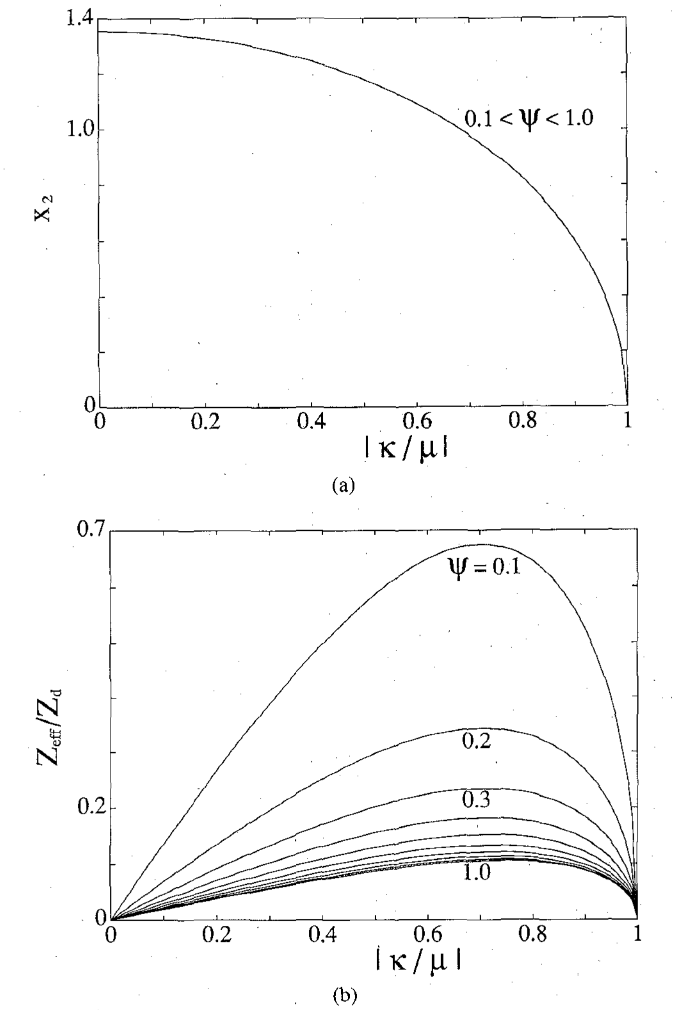


Fig. 4. Circulation conditions of the Y ring with $s = 0.5$. (a) First condition. (b) Second condition.

and 4 respectively. These can be displayed in the usual way, i.e., for the first condition a graph of $x_2 = k_{\text{eff}}R$ versus κ/μ, and for the second condition Z_{eff}/Z_d vs. κ/μ. The actual impedance ratio [1] is given by

$$\frac{Z_{\text{eff}}}{Z_d} = \sqrt{\frac{\epsilon_d \mu_{\text{eff}}}{\epsilon_f}} = \sqrt{\frac{\epsilon_d}{\epsilon_f}\left(1 - \left(\frac{f_m}{f}\right)^2\right)} \tag{24}$$

If (24) is superimposed on the graph of the second circulation condition the intersection of the two curves determines the circulation frequency.

To predict the performance of a symmetrical Y ring circulator and compare it with that of a Y disk, the TT2-2750 Nickel ferrite [17] with $4\pi M_s = 2750$ Gauss and $\epsilon_f = 12.8$ was chosen as suitable for the frequency range of interest. A medium of dielectric constant $\epsilon_d = 10$ was selected to surround the ferrite. Using the above mentioned data, tracking circulation in the symmetrical Y disk circulator occurs when $\psi = 0.572$ over the range $0.7 \leq \kappa/\mu \leq 1.0$. Using (22) the magnetization frequency $f_m = 7.7$ GHz and hence from (20) and (21) the circulation frequency is $f_c = f_m/0.7 = 11$ GHz. Using (24) $Z_{\text{eff}}/Z_d = 0.631$. From circulation conditions for the disk [1] ($s = 0$) and for the several ring circulators, Table I shows the ψ values, the corresponding R values and the strip widths of both the disk and ring circulators operating at 11 GHz. The ψ values were calculated from the second circulation conditions using the method of interpolation and extrapolation and the R values from the first circulation conditions using

$$R = \frac{x_2}{\dfrac{2\pi f_m \sqrt{\epsilon_f}}{c} \dfrac{\sqrt{1 - \left|\dfrac{\kappa}{\mu}\right|^2}}{\left|\dfrac{\kappa}{\mu}\right|}} \quad \text{for } f_o \ll f. \tag{25}$$

For values of s larger than 0.7 it becomes very difficult to satisfy the circulation condition.

With these dimensions the performance of each circulator was obtained by calculating the elements of the scattering matrix $[S]$ using the following transformation

$$[S] = \frac{[Z] - [I]}{[Z] + [I]} \tag{26}$$

TABLE I
Y-JUNCTION DISK AND RING CIRCULATOR DIMENSIONS FOR 11 GHz OPERATION

$s = R_{in}/R \rightarrow$	0	0.1	0.2	0.3	0.4	0.5
R (mm) $\rightarrow$	2.164	2.143	2.107	1.996	1.816	1.663
ψ (rad) $\rightarrow$	0.572	0.551	0.484	0.347	0.205	0.107
W (mm) $\rightarrow$	2.341	2.243	1.961	1.357	0.738	0.355

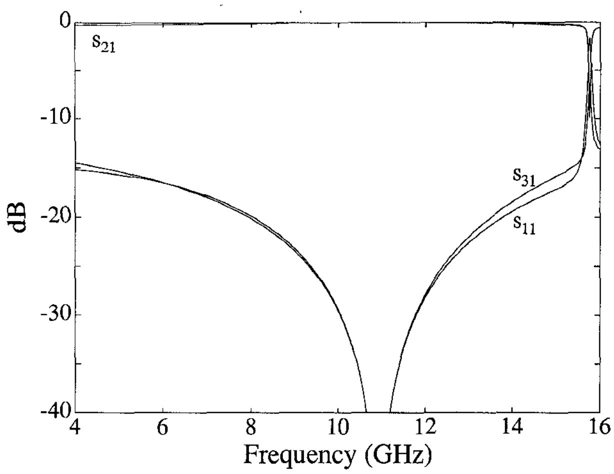


Fig. 5. Predicted performance of intrinsic disk circulator, $s = 0$. Parameters given in text.

in which $[Z]$ is the impedance matrix whose elements for the disk given in (7) and (8) and for the ring given in (5) and (6) and $[I]$ is the 3×3 unity matrix. Figs. 5, 6, and 7 show the reflection coefficient s_{11}, the isolation coefficient s_{31}, and the transmission coefficient s_{21}, of the disk, and the $s = 0.2$ and $s = 0.5$ rings, respectively for the range 4 to 16 GHz. It can be seen that the disk exhibits the widest bandwidth. The insertion loss spike that occurs with the disk, due to the $n = \pm 2$ terms in the field expansion, becomes wider with the $s = 0.2$ ring, and very wide with the $s = 0.5$ ring.

If a typical value of the microstrip dielectric substrate of $h = 1$ mm is chosen, the characteristic impedance values Z_o of the disk and ring circulators can then be calculated using [15] and the values of W given in Table I. Table II shows these Z_o values and the 20 dB bandwidths of both disk and ring circulators.

From Table II it can easily be seen that the characteristic impedance Z_o, for $h = 1$ mm, increases as s increases, and when $s = 0.3$ $Z_o = 41.29\ \Omega$ and when $s = 0.4$ $Z_o = 55.72\ \Omega$, therefore between $s = 0.3$ and $s = 0.4$ there must exist a value of s which corresponds to $Z_o = 50\Omega$. After a search in the derived circulation conditions for values of $0.3 < s < 0.4$ with increments of 0.01 it was found that the ring with s = 0.36 operating at 11 GHz has a radius $R = 1.887$ mm, a coupling half-angle $\psi = 0.257$ rad and a characteristic impedance $Z_o = 49.35 \approx 50\Omega$ and exhibits a 20-dB isolation bandwidth of 1.05 GHz (Fig. 8).

If a Y disk circulator is required without transformers, and the value of the characteristic impedance required at

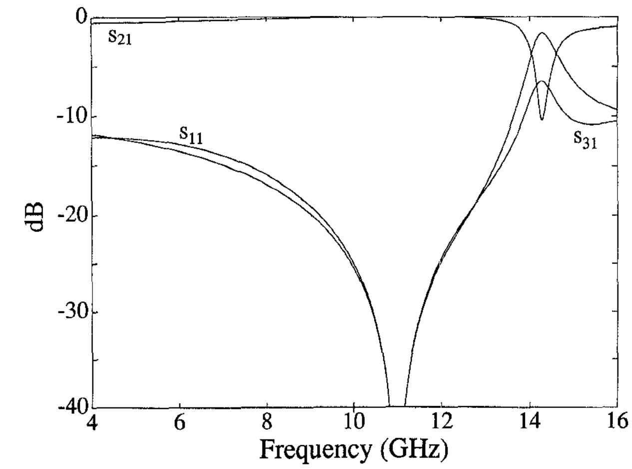


Fig. 6. Predicted performance of intrinsic ring circulator, $s = 0.2$. Parameters given in text.

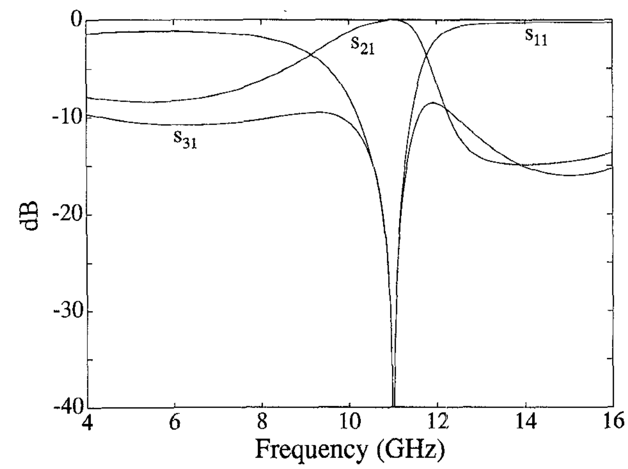


Fig. 7. Predicted performance of intrinsic ring circulator, $s = 0.5$. Parameters given in text.

TABLE II
Z_o AND BANDWIDTH OF BOTH DISK AND RING CIRCULATORS

s ratio $\rightarrow$	0	0.1	0.2	0.3	0.4	0.5
$Z_o(\Omega) \rightarrow$	29.86	30.70	33.40	41.29	55.72	74.36
B.W. (GHz) $\rightarrow$	5.35	4.78	3.39	1.52	0.74	0.39

the disk edge is $Z_o = 50\ \Omega$, it can be shown from [15] that $W/h = 0.956$ and hence it follows that impedance transformers can be avoided by selecting $h = 2.45$ mm.

The choice of a very thick substrate may in general lead to the excitation of higher order modes which would influence the performance of the Y disk circulator. However, the onset of higher order modes could not occur at wavelengths greater than [16]:

$$\lambda_o = \frac{H\sqrt{\epsilon_r - 1}}{0.354} \tag{27}$$

With $\epsilon_r = 10$ and $h = 2.45$ mm, $\lambda_o = 20.75$ mm, i.e., the cut-off frequency for the lowest order mode is $f = 14.46$ GHz. From Fig. 5 it can be seen that at 20 dB iso-

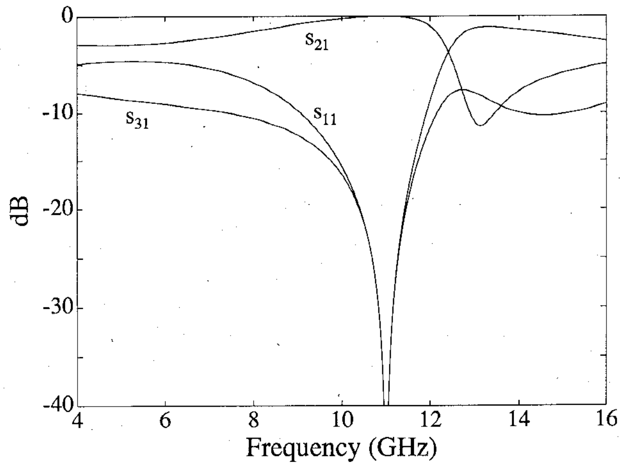


Fig. 8. Predicted performance of intrinsic ring circulator, $s = 0.36$. Parameters given in text.

lation $f_{\max} = 13.56$ GHz, hence $f_{\max} < f$ and therefore the choice of $h = 2.45$ mm is permissible.

Figs. 5, 6, 7, and 8 do not necessarily represent optimum performance and further work is in hand.

Asymmetrical W Disk Circulators

The newly proposed W stripline disk circulator is shown in Fig. 9. The conditions for circulation from port 1 to port 2 and from port 3 to port 1 are identical and are shown in Fig. 10(a, b). The conditions for circulation from port 2 to port 3 are shown in Fig. 11(a) and (b). Circulation conditions for transmission 2 → 3 and 3 → 1 resemble (2) and (3) with Z_{in_2} and Z_{in_3} in place of Z_{in_1}.

Z_{in_2} and Z_{in_3} are given by

$$Z_{\text{in}_2} = Z_{22} - \frac{Z_{12}Z_{23}}{Z_{13}} \tag{28}$$

and

$$Z_{\text{in}_3} = Z_{33} - \frac{Z_{31}Z_{23}}{Z_{21}}. \tag{29}$$

The impedance matrix elements of the W-disk circulator are given in (7) and (8) with setting $\phi_1 = 0$, $\phi_2 = \pi/3$, $\phi_3 = 2\pi/3$, $\psi_1 = \psi_2 = \psi_3 = \psi$. The maximum coupling angle allowed for the W-disk is $\psi_{\max} = \pi/6 = 0.52$ rad. As can be seen from Figs. 10(b) and 11(b) the impedance ratios are negative and so circulation will occur in the opposite sense from that indicated in Fig. 9. These Mode 1 circulation conditions are restricted to $|\kappa/\mu| < 0.5$, and therefore the circulation frequency must be at least $2f_m$.

To explore the predicted performance of these W-circulators, a ferrite with $4\pi M_s = 5000$ G and $\epsilon_f = 12.5$ was chosen. The surrounding dielectric medium was selected to be quartz, $\epsilon_d = 4.4$, and the circulation frequency 94 GHz. The design procedure described above was followed with the two sets of circulation conditions shown in Figs. 10 and 11, and each resulted in different dimensional data, R and ψ, as listed in Table III, together with W from (1). The predicted performance of design A is

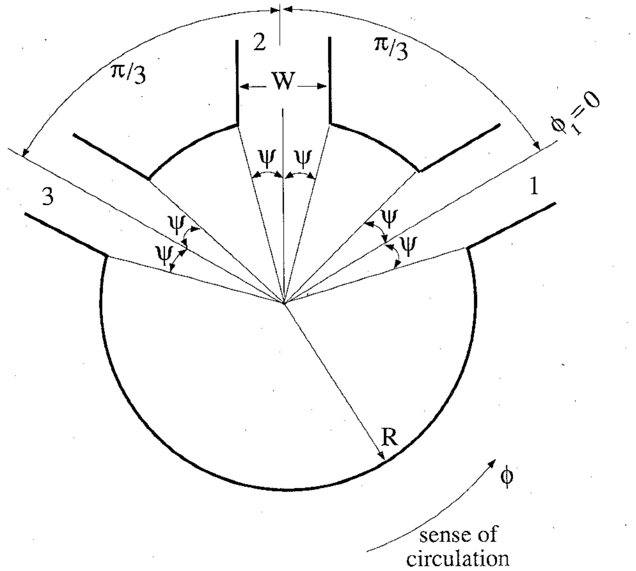


Fig. 9. Coordinate system for a ferrite W disk circulator.

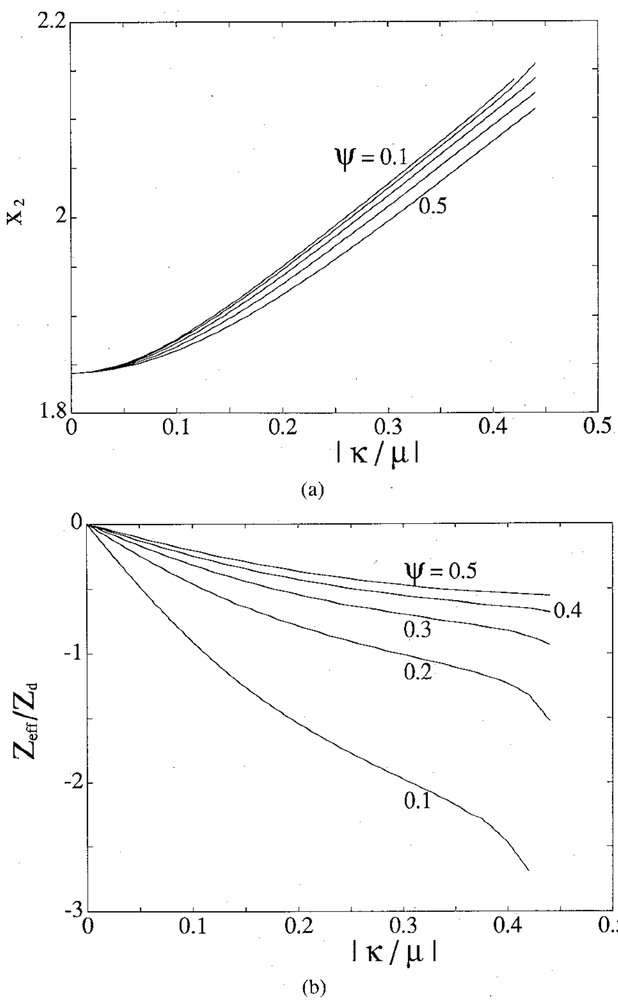
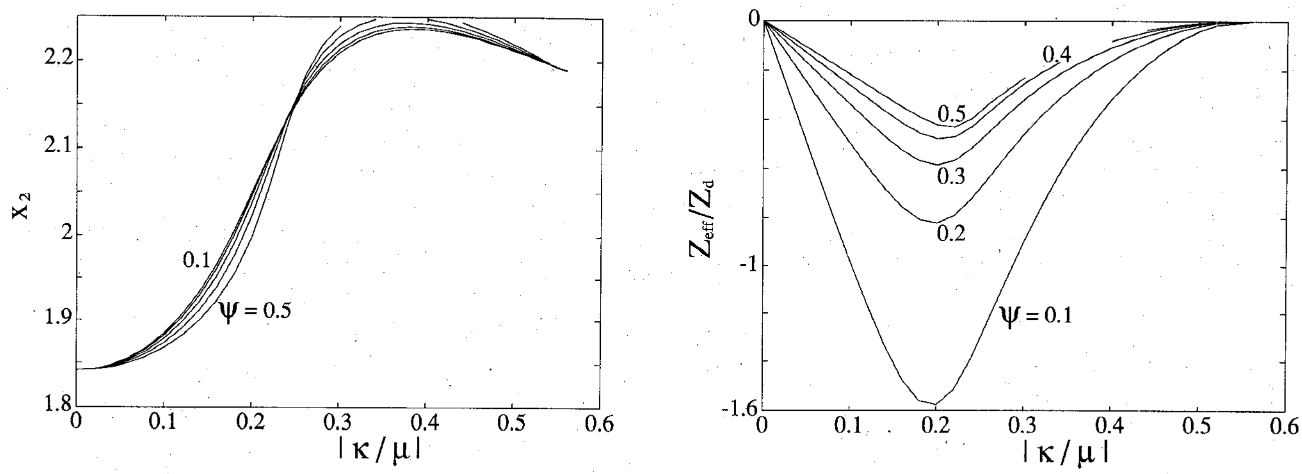


Fig. 10. W-disk Conditions for circulation from ports 1 to 2 and ports 3 to 1. (a) First condition. (b) Second condition.

shown in Fig. 12(a) and of design B is shown in Fig. 12(b), with the input at each port in turn. It can be seen that the insertion loss, isolation and reflection loss are well

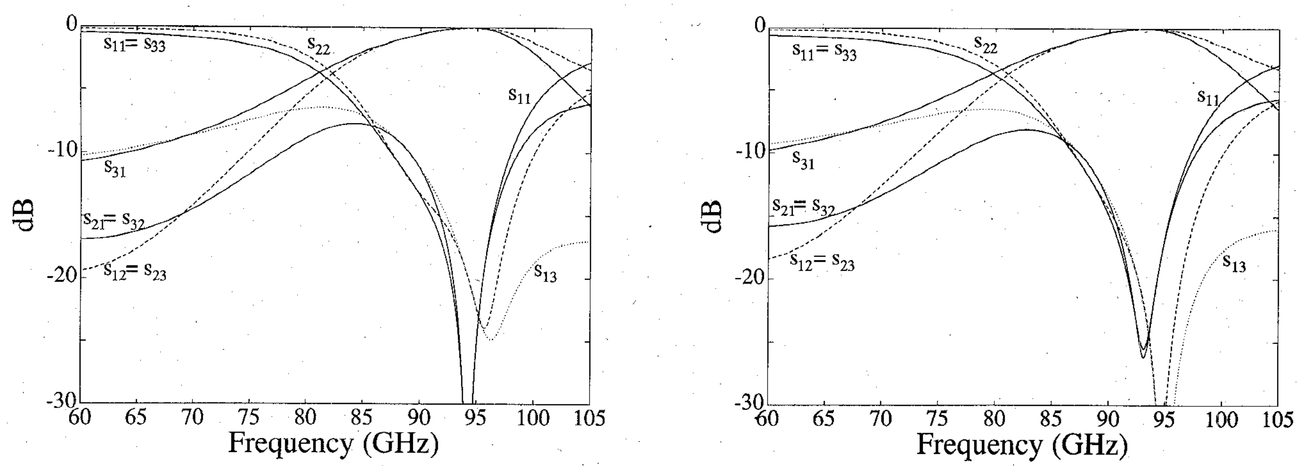
TABLE III
DISK DATA FOR THE W DISK

Design A based on 1 → 2 Conditions	
R (mm) →	0.277
ψ (rad) →	0.219
W (mm) →	0.120
Design B based on 2 → 3 Conditions	
R (mm) →	0.281
ψ (rad) →	0.248
W (mm) →	0.138



(a) (b)

Fig. 11. W-disk Conditions for circulation from ports 2 to 3. (a) First condition. (b) Second condition.



(a) (b)

Fig. 12. Predicted performance of an intrinsic 94 GHz W-disk circulator. (a) Using conditions in Fig. 10(a) and (b). (b) Using conditions in Fig. 11(a) and (b). Design parameters given in text.

aligned when the input is at port 1 and the output at port 3. When the input is at port 2 or port 3, the reflection loss or the isolation is misaligned (off frequency). It is not known yet whether the design parameters can be adjusted to produce improved frequency alignment. Selection of the substrate thickness will determine whether transformers to 50 Ω lines are required.

CONCLUSION

The circulation conditions for a ferrite ring Y circulator have been discussed. The presence of a hole in the middle of the disk reduces the bandwidth, but it enables smaller coupling angles to be used and transformers to be eliminated. With the ratio $s = R_{in}/R$ larger than 0.7 it is difficult to satisfy the circulation conditions, and for practical purposes $s \leq 0.5$ is preferred. A design with $s = 0.36$ has been given for operation at 11 GHz with a 20-dB isolation 1 GHz bandwidth, with 50 Ω feeder lines, and no transformers.

The circulation conditions for an asymmetrical "W-circulator" have been presented for the first time. The circulation conditions depend on the selected input port and the direction of circulation. The Mode 1 circulation conditions are restricted to $|\kappa/\mu| < 0.5$, and therefore the circulation frequency must be greater than $2f_m$. Two designs for operation at 94 GHz have been discussed, based on circulation from port 1 to port 2 and from port 2 to port 3, and the predicted behavior for each case has been discussed. It is not yet clear to what extent small misalignments in the frequency response of the s-parameters can be reduced by design modifications, and further work is in hand.

Both of these new circulator structures may offer advantages over disk circulators in systems where narrow bandwidths are acceptable and compact designs are required.

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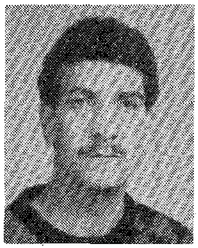

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